An Airborne Experimental Test Platform

From Theory to *Flight* (Part 2)

Three airframes used by the UMN-URG. Multiple UAVs can use cooperative navigation to improve navigation performance.

Advances in guidance, navigation and control technology make it possible to successfully use UAVs in a growing range of applications. The University of Minnesota UAV Research Group uses UAVs as test platforms to research and develop GNC avionics. This safe, cost effective research would not be possible with manned aircraft or simulation-only analysis. In this second article of a two-part series, researchers at UMN-URG illustrate how they use UAV test platforms to develop, test and certify new avionics and GNC algorithms for safety-critical systems.

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Although most UAV applications to date have appeared in the military and law enforcement sectors, many important civilian applications exist for these vehicles. Such uses include wildfire surveys, transportation infrastructure inspection, and precision agriculture. Recent advances in guidance, navigation and control (GNC) technology have made these civilian applications possible.

Furthermore, UAVs provide an excellent surrogate platform for researching and developing avionics and GNC algorithms intended for the safety-critical application of guiding, navigating and controlling manned aircraft. This not only offers significant cost savings, but testing experimental avionics and algorithms on a UAV also poses less risk.

UAVs have been used as such test platforms before. NASA Armstrong (formerly Dryden) flight research center, for example, has used X-48 and X-56 to test experimental control laws that are too risky to test on manned aircraft.

The University of Minnesota UAV Research Group (UMN-URG) focuses on researching, developing, and testing advanced avionics algorithms using low cost UAVs as test platforms. The UMN-URG's platform allows for safe and cost-effective GNC research that would be impossible with manned aircraft or with simulation-only analysis.

In Part 1 of this series, which appeared in the March/April 2014 issue of *Inside GNSS*, we described the open flight research platform used for this work, including the experimental aircraft, simulation architecture, and flight software algorithms. In this follow-on article, we present specific examples that highlight how UAVs can be employed in GNC avionics research and development. First, we will describe a GNSS-enabled air data estimation technique whereby an aircraft's equation of motion is used as a virtual sensor to aid in estimating its airspeed, angle of attack, and sideslip angle.

Next, we discuss the development of navigation algorithms for GNSS-stressed or GNSS-denied environments highlighting how information sharing among UAVs flying in a certain region could be used to coast through GNSS outages. Finally, we present work that focuses on how UAVs can be used to assess the reliability of an avionics system, including certification tools and fault detection algorithms.

GNSS-Enabled Air Data Estimation

An air data system provides information about an aircraft's speed and orientation (angle of attack and sideslip angle) relative to the outside air. Estimates of airspeed, angle of attack, and sideslip angle are crucial for safely and efficiently operating an aircraft. For example, airspeed is used to define the maximum speed beyond which structural damage can occur. Angle of attack and sideslip angle are used to ensure an airplane does not operate in a region from which recovery from an upset would be impossible.

An air data system is composed of several components. The main ones are a pitot-static system that measures the dynamic and static air pressure, and aerodynamic vanes that measure angle of attack and sideslip angle. These measurements are then processed by an air data computer that refines the air data estimates with temperature and local flow corrections.

One well-known fault mode to this airspeed measuring method is ice accumulation on probe pressure inlets. This causes obstruction that leads to erroneous pressure measurement. As shown in **Table 1**, ice accretion on air data sensors has been involved in numerous accidents/incidents over the years.

Heating elements are now commonly used on commercial transport aircraft to prevent ice accumulation and probe inlet clogging. Recent climate studies show changes in upper atmosphere weather patterns, which portend more frequent icing encounters such as that which caused the accident of Air France Flight 447.

Date	Aircraft	Precipitating Event	Results
Dec. 1, 1974	B727, Northwest 6231	Pitot icing	Total fatalities
Feb. 6, 1996	B757, Birgen Air 301	Pitot tube blockage (insect)	Total fatalities
Oct. 2, 1996	B757, Aero Peru 603	Static port blockage (tape)	Total fatalities
May 12, 2005	B717, Midwest 490	Pitot icing	Diverted, landed safely
Feb. 5, 2007	A330, Qantas 72	Air data spikes	12 serious injuries, 113 minor injuries
Nov. 27, 2008	A320, XL Air 888T	Stuck AoA vane (ice)	Total fatalities, hull loss
June 1, 2009	A330, Air France 447	Pitot icing	Total fatalities, hull loss
June 23, 2009	A330, Northwest 8	Pitot icing	No damage, landed safely
Oct. 28, 2009	A330, Jetstar 12	Temporary pitot icing	Incident, no damage

TABLE 1. Several incidents/accidents that involve faulty air data systems

These concerns have led the aviation industry and government regulatory agencies to ask: "Is there a way to determine aircraft speed and orientation that is immune to ice buildup and can be easily retrofitted into existing aircraft?"

The UMN-URG has developed a method to synthetically estimate these air data quantities using a GNSS receiver, a MEMS inertial measurement unit (IMU), and a mathematical model of the aircraft. We call this method synthetic because it doesn't rely on direct pressure or aerodynamic angle measurements to estimate the air data quantities.

GNSS receivers and IMUs have become part of the standard aircraft instrumentation. They enable most, if not all, navigation systems these days; so, being able to use these sensors to estimate air data quantities is very attractive.

Although our method requires instrumenting the aircraft with sensors that can measure the control surface deflections and propulsive force in flights, this is not a very demanding requirement. This information is available on most aircraft equipped with an autopilot unit, especially if servos are used to actuate the flight control surfaces.

Our approach to this problem uses a federated extended Kalman filter (EKF) architecture where two estimators are cascaded in series. The functional block diagram of this architecture is shown in **Figure 1**. This architecture is completely independent of the pitot-static-vane system and is formulated in the filtering framework that allows us to estimate the accuracy of the synthetic estimate of airspeed (V), angle of attack (α), and sideslip angle (β) through the filter error covariance matrix.

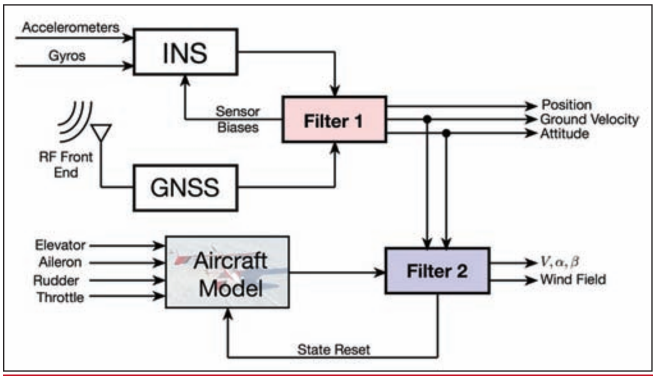

FIGURE 1 Synthetic air data system architecture

This algorithm was tested on the UMN-URG's Ultrastick 120, known as Ibis. The synthetic air data estimates were then compared with measurements made using a calibrated pitot tube and angle of attack and sideslip vanes.

Figure 2 shows the filter performance on one of Ibis's flights, and we can see that

the estimation errors are bounded by the estimate of the 3-σ values. In the nominal flight condition, the accuracy (3-σ) of the synthetic airspeed, angle of attack, and sideslip estimates are less than two meters/second, three degrees, and five degrees, respectively. We could improve these results with a more rigorous system identification process to build a nonlinear aircraft model.

Future Navigation Concepts

The FAA Modernization and Reform Act of 2012 (HR658) requires the Federal Aviation Administration (FAA) to integrate routine unmanned aircraft operations into the national airspace system (NAS). Section 332 of the legislation requires the FAA to "provide for the safe integration of civil unmanned aircraft systems into the national airspace system as soon as practicable, but not later than September 30, 2015."

The term "safe" is interpreted to mean that UAVs must possess an equivalent level of safety as manned aircraft or must not pose undue hazard to other aircraft or the general public in the vicinity. This implies that integrating UAVs into the NAS must be seamless and that they must be able to operate side by side with manned aircraft.

In this future airspace, GNSS, in particular GPS, will be the key technology used for traffic separation and monitoring. In some of the envisioned concepts, aircraft will self-report their position (by means of automatic dependent surveillance, or ADS-B, technology) with this information being used for air traffic control purposes. In this application, an incorrect or false position report, regardless of whether it was intentional or malicious, could have severe consequences.

The UMN-URG has developed a civilian GPS authentication approach that can deal with this problem. The system works by validating the position report sent by each aircraft against a segment of the GPS signal collected by an authenticator (e.g., air traffic controller). Our experiment shows that this authentication methodology's resolution is better than 15 meters.

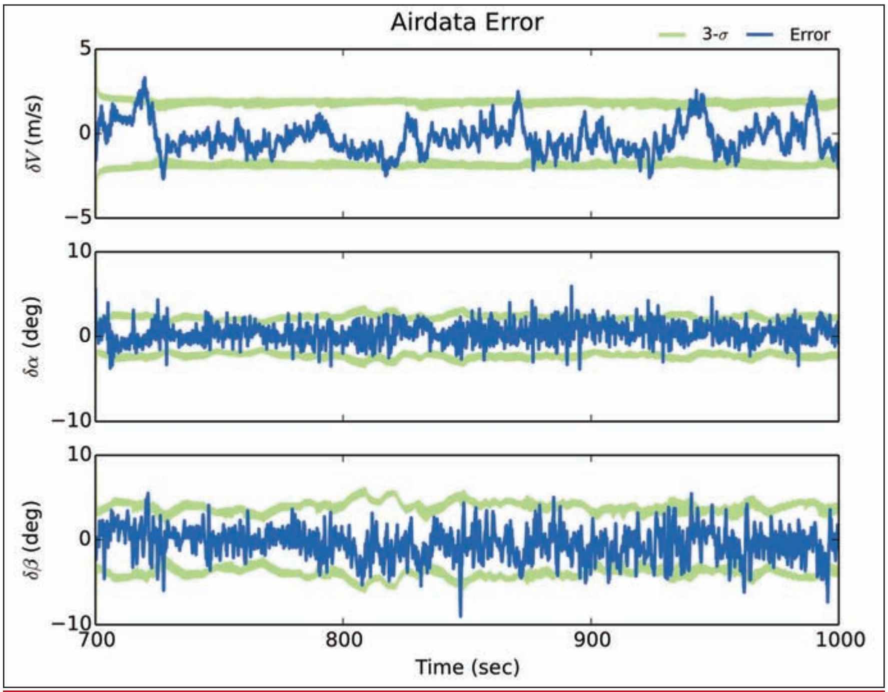

FIGURE 2 Synthetic air data estimation error for Ibis Flight 26

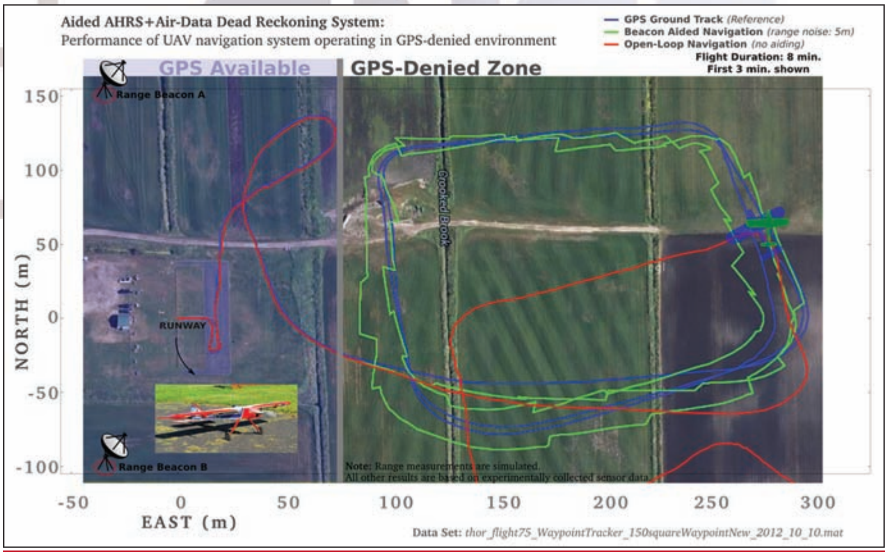

FIGURE 3 Demonstration of navigation capability of air-data based dead reckoning system aided by range measurements to two beacons

A "swarm" or community of several UAVs operated simultaneously can provide a miniature replica of the future airspace in which one can test such authentication techniques. A community of UAVs can also be used to evaluate information sharing–based navigation concepts that have been envisioned as part of many alternative positioning and timing (APNT) systems for dealing with GNSS-stressed or denied scenarios.

Traffic collision avoidance systems (TCAS) and ADS-B are examples of existing systems that allow such information sharing. Received by modem-equipped neighboring vehicles and coupled with relative measurements such as *range* between the vehicles, this information can serve as an aiding source.

Although TCAS probably won't be used on small UAVs, sensors providing relative measurements such as *range* or *bearing*, are anticipated to become commonly available in the near future as

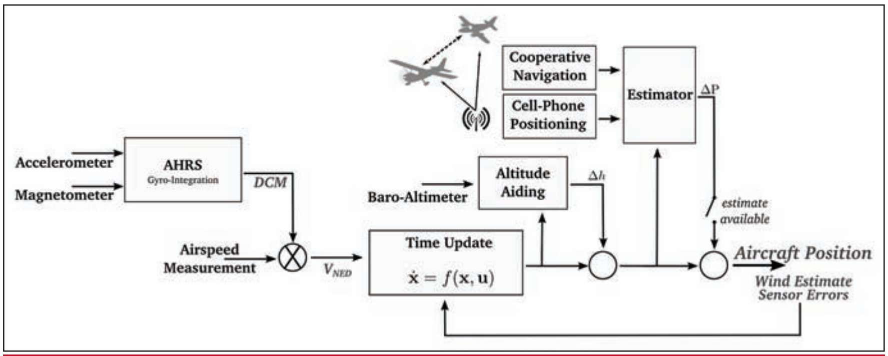

FIGURE 4 A schematic of the estimation scheme for GNSS-denied navigation using air data dead-reckoning aided by cell-phone and cooperative navigation

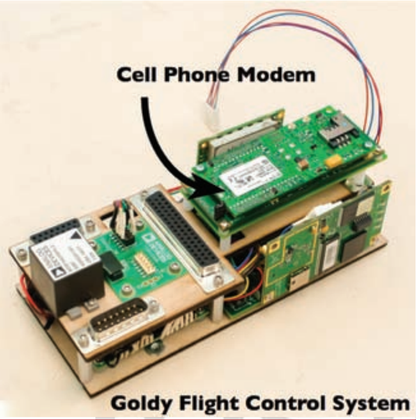

FIGURE 5 Goldy flight control system augmented with a cell phone modem

aviation authorities implement requirements for detect-and-avoid capabilities. Furthermore, a data link such as the 900MHz modem can be used to exchange information with a ground station. Therefore, it is not difficult to imagine neighboring vehicles effectively acting like moving beacons that can be used for navigation purposes, improving the positioning accuracy of all vehicles in the area.

Information from aircraft equipped with higher grade navigation equipment can also be shared with other vehicles. While cooperative navigation is useful in general, it is especially helpful for operations in geographic areas experiencing denial (e.g., jamming, blocking, interference) of GNSS signals.

With cooperative navigation, the rate of error growth among vehicles operating in the GNSS-denied zone can be significantly reduced. Drift-free solutions of vehicles operating outside the GNSS-denied zone can be propagated to those affected by the GNSS-denial. This can improve robustness for operating in urban areas or overcoming other sources of GNSS signal interference.

Aiding in the form of inter-vehicle measurements and information exchange introduces a correlation between the state errors of the vehicles. Handling this correlation is the major challenge in making cooperative navigation a reality. The UMN-URG has developed an effective algorithm for accomplishing this, which has been tested in post process using flight test data.

Cooperative navigation is not the only GNSS-denied navigation solution that UMN-URG is considering. A cell-phone signal aided dead-reckoning navigator, as shown in **Figure 4**, is another alternative. **Figure 5** shows how the cell phone modem will be integrated into the Goldy flight control system (described in the first article of this series). This is the subject of current and ongoing work and beyond the scope of this article.

Cooperative Navigation Flight Tests

A set of seven flights collected between 2011 and 2013 have been aggregated and used for a cooperative navigation analysis. The ground-track of each flight is shown in **Figure 6**. During the flights, each UAV had access to the following information: three-axis accelerometer, gyro, and magnetometer measurements, GPS position and velocity, airspeed, and baro-altimeter measurements. (These data were also used to implement the AHRS and dead-reckoning system schematically shown in Figure 4.)

Artificial inter-vehicle range measurements were derived using the GPS measurements. Using real flight data, our research demonstrates the feasibility and attainable performance for a community of vehicles navigating in GNSS-denied environments with the aid of cooperative navigation.

Using the collected flight data from the UAVs, we employed post-processing analysis to determine unaided performance when GPS is unavailable. All seven flights were temporally shifted so as to occur simultaneously. At time t = 110 s, GPS was denied to all aircraft for two minutes. During this time, each aircraft continued to rely on the on-board dead-reckoning system.

The light blue line in **Figure 7** shows about 450 meters of uncertainty in the position estimate by the end of the two minutes. Although the data in Figure 7 are particular to Thor Flight 75, similar figures could be generated for the other six UAV flights.

When cooperative navigation is used, however, the final uncertainty after two minutes of GPS outage is only 150 meters, as indicated by the green line in Figure 7. Note that this benefit is entirely due to the inter-aircraft ranging and information exchange, because none of the aircraft has access to GPS. Had even one aircraft had access to GPS or a high-grade inertial navigation system, the navigation capabilities of the entire community would have been significantly improved.

The UMN-URG's UAV platform and online archival of flight data has served as an enabling tool for cooperative navigation estimator design and validation. Although current development work uses playback of logged flight sensor data, we hope to use the multiple UAV platforms to demonstrate real-time cooperative navigation in the future.

Assessing System Reliability

Actuator and sensor failures are two main causes of major aircraft accidents. A famous example of actuator failure is uncommanded rudder movement encountered on Boeing 737s, that was

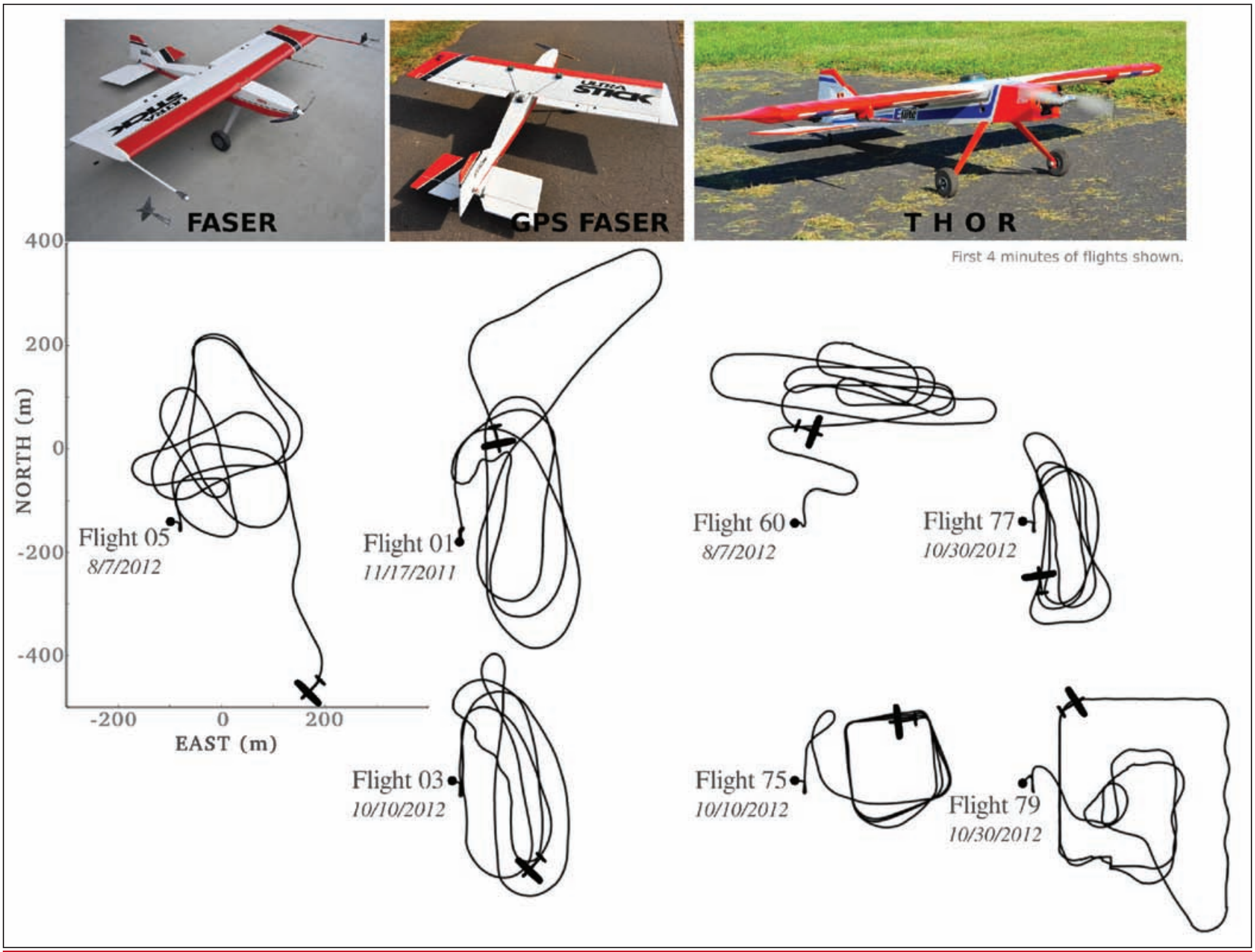

FIGURE 6 Ground track of seven flights used for cooperative navigation research. Flights occurred at the same airfield (coincident •), but are spatially shifted for visual clarity. Collaborative navigation analysis done using temporally shifted flights to occur concurrently.

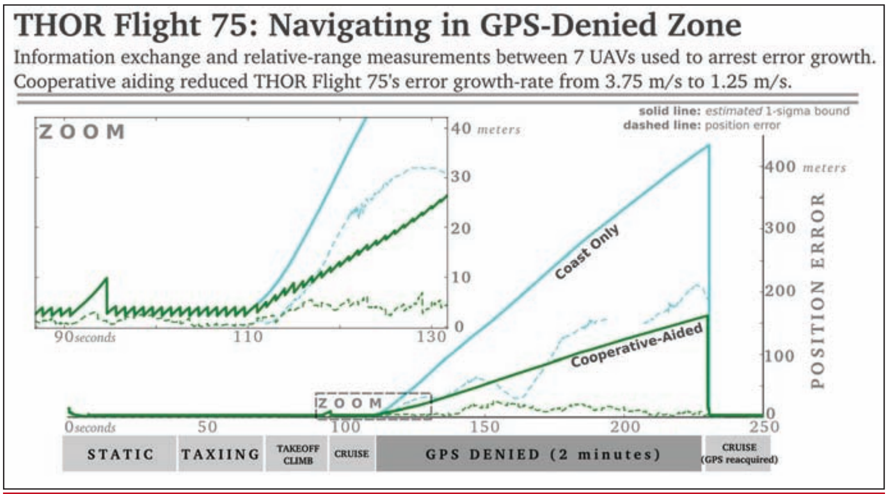

FIGURE 7 Time evolution of position error for THOR Flight 75 navigating in GPS-denied area

believed to have been the cause of accidents in 1992.

Sensor failure, however, is more common. Although it doesn't cause loss of control directly, sensor failure can lead to loss of situational awareness. Faults in an air data system is an example of sensor failure.

UAVs are ideal surrogate platforms to test algorithms that detect and isolate system faults because of the risk these tests pose on manned aircraft. The UMN-URG is using UAVs to help develop robust fault detection and isolation algorithms, as well as a way of certifying these algorithms. Eventually, these could be used to inform airframe design changes to improve fault tolerance.

Control Surface Impairments. In fall of 2013, as part of an undergraduate

senior design project, a group of students assessed the reliability of the existing (baseline) UltraStick 120 used by the UMN-URG.

The team performed a *failure modes and effects analysis (FMEA)* as well as a *fault tree analysis (FTA)*. Through this analysis, the team estimated the existing Ultra Stick 120's catastrophic failure rate as 2.17 failures per 100 hours. This is a type of failures where the aircraft's controls are critically altered and emergency landing with moderate to high damage is likely. Some examples include loss of radio control or a stuck control surface that leads to an aircraft that is not trimmable.

The team determined the reliability could be improved to 0.128 catastrophic failures per 100 hours with a few simple designs updates. In particular, the team recommended the use of split rudder/elevator surfaces (See **Figure 8**), as well as a redundant battery and redundant failsafe switch.

The students are currently building the re-designed Ultra Stick 120 that incorporates some of these changes, including the split rudder/elevator surfaces. This redesigned Ultra Stick 120 will serve as the reliability platform for the UMN-URG in the near future.

Fault Detection and Identification (FDI). Reliability and safety requirements for commercial manned flight control electronics are on the order of no more than 10^{-9} catastrophic failures per flight hour. Commercial aircraft meet this stringent reliability requirement via hardware redundancy throughout the flight control system.

For example, the Boeing 777 flight control electronics consist of three primary flight computing modules that each contains three dissimilar processors. The actuators and sensors have similar levels of redundancy. This configuration allows the system to isolate failures so no single event or component failure can cause the entire aircraft system to fail.

The UMN-URG is investigating model-based analytical redundancy as an alternative to achieve fault tolerance. This approach relies on mathematical models and/or measured data to detect and identify faults instead of additional hardware to ensure against them. The model-based algorithm was developed using a mathematical model of the aircraft, while the data-driven algorithm operates exclusively on raw flight test data.

We devised UAV flight tests with faulted and unfaulted aileron actuators to acquire telemetry data. We subsequently assessed detection performance by playing back the experimental flight data and applying both detection algorithms. The main contribution of this work is its demonstration that experimental data allows for a side-by-side comparison of FDI techniques arising from different philosophies of system health monitoring.

Figure 9 shows results from several experimental flight tests where faults were purposefully injected into the system. In these tests, we considered aileron fault (f_a) as well as rate roll rate gyro fault (f_p). Our FDI algorithm was able to accurately estimate two faults that occurred at different times in flight.

Certification Tools. Certification is another key issue that the UMN-URG is investigating. Specifically, aircraft designers need to certify the reliability of an analytically redundant system with aviation authorities, e.g., the Federal Aviation Administration (FAA) in the United States or the European Aviation Safety Agency (EASA). The system must not only be highly reliable and safe, but it must also possible to certify the system's reliability and safety.

In a *physically redundant configuration,* a failed component is detected by directly comparing the behavior of each redundant component. Hence, these architectures tend to detect

FIGURE 8 Split Rudder Design on Ultra Stick 120

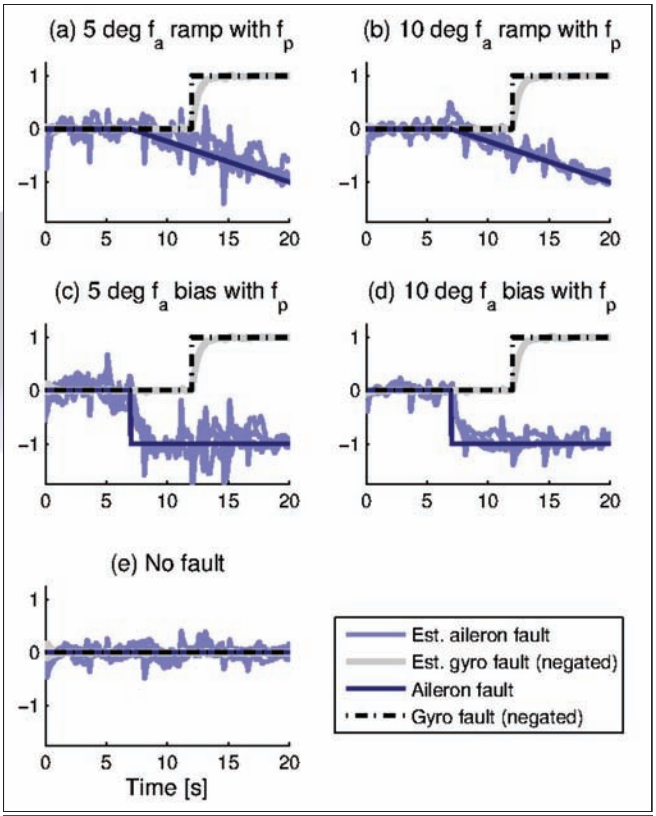

FIGURE 9 Experimental model-based FDI results. Full 5-input filter estimates with normalized faults and gyro faults (gyro fault and estimates negated for clarity)

faults accurately and quickly. Their performance can be certified from known hardware component failure rates using FMEA and FTE methodology.

The reliability of systems that use *analytical redundancy*, on the other hand, depends on the performance of the detection algorithm as well as the hardware component failure rates. However, with the latter method new failure modes are introduced due to the mixed use of analytical algorithms and hardware components. Thus, different tools are required to assess the reliability of analytically redundant systems.

Over the years, the UMN-URG has developed analytical tools to assess the reliability of analytically redundant systems. For example, we derived a theoretical method to assess the probabilistic performance for a dual redundant system as shown in **Figure 10**. Applying this method, we compute the system failure rate per hour based on knowledge of the failure rates for the hardware components and, for the FDI logic, knowledge of specific probabilistic performance metrics. This probabilistic method can complement the use of Monte Carlo simulations when evaluating the system's reliability. Although this method is faster and more efficient, it can only be applied given sufficient information about the hardware components and the FDI algorithms.

Conclusion

Renewed interest has emerged for developing new avionics and GNC algorithms for safety-critical systems. The UMN-URG's research focuses on developing, testing and certifying such algorithms using low-cost UAVs as test platforms. In particular, we develop multisensor navigation and estimation algorithms, fault detection and isolation algorithms, and system reliability assessment and certification tools. We use UAVs to test them.

The open-source infrastructure encourages collaboration with the entire research community, especially through sharing flight data. All information on the research platforms, including archived flight, data are available on the UMN-URG's website <http://www.uav.aem.umn.edu>.

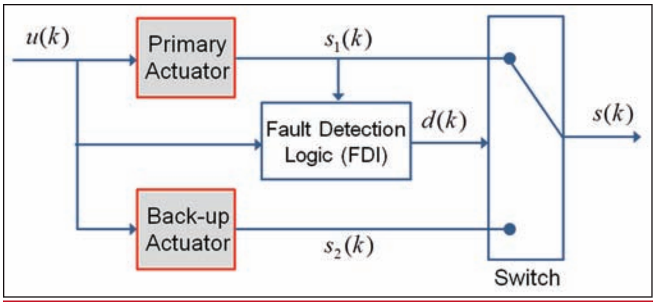

FIGURE 10 A dual-redundant system architecture with fault detection logic

Acknowledgment

The work described in this article was supported by the United States Department of Homeland Security (DHS) for supporting this work through the National Center for Border Security and Immigration under grant number 2008-ST-061-BS0002, the National Science Foundation under Grant No. 0931931 titled "CPS: Embedded Fault Detection for Low-Cost, Safety-Critical Systems," and NASA under Grant No. NRA NNX12AM55A titled ``Analytical Validation Tools for Safety Critical Systems Under Loss-of-Control Conditions."

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the DHS, the National Science Foundation, or NASA. We also want to acknowledge the members of the senior design team: Jeremy Amos, Erik Bergquist, Jay Cole, Justin Phillips, Shawn Reimann, and Simon Shuster.

Additional Resources

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Peter Seiler is an assistant professor in the Department of Aerospace Engineering and Mechanics, University of Minnesota, Twin Cities. In his research into control systems with applications to

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Gary Balas is a professor and the department head of the Department of Aerospace Engineering and Mechanics, University of Minnesota, Twin Cities. His main research interest is in narrowing the gap between engineering requirements, real-time control implementation, and theoretical control analysis and design techniques. He has focused on describing "real" physical systems via sets of models with the goal of developing an integrated framework for fault detection and isolation, control modeling, analysis, and synthesis, based on physically motivated assumptions, which make use of the specific characteristics of each system to be controlled. IG

GNSS Solutions *continued from page 39*

using standalone positioning techniques, the resulting error will almost always be less than 100 meters.

Even if one pessimistically assumes the base station error and the inter-receiver distance are both one kilometer, the second term in equation (9) has a maximum possible value of about five centimeters [= (1 km)^2 / 20,000 km], which is quite small (at least for pseudorange measurements).

Where the effect of base station error can become a problem is when very large inter-receiver distances are considered. A few examples of this are included in the M.Sc. thesis by C. Tang ("Accuracy and Reliability of Various DGPS Approaches", May 1996, University of Calgary, UCGE Report No. 20095, available at <http://www.geomatics.ucalgary.ca/graduatetheses>).

Summary

The key takeaway from this discussion is that differential processing of GNSS data ultimately produces an estimate of the relative position of the two receivers involved. Only if the base station coordinates are known in an absolute sense are the coordinates of user receivers also absolute in nature.

Another point worth noting is that although the foregoing mathematical development focused on the use of pseudorange measurements, the same development also applies to carrier phase data. The main difference in the latter case is that the carrier phase ambiguity terms need to be included. Also, because the carrier phase measurement errors are smaller than those of the pseudorange (in terms of noise and multipath), carrier phase processing is a bit more sensitive to base station positioning errors.

We should also note that, even though carrier phase process often uses double differencing techniques, the between-satellite difference does not negate any of the above development. IG